Online Science Olympiads

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International Physics Online Olympiad

2014 World Cup

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Instructions

Enclosed are 5 multiple-choice questions, 10 short-response questions, and 5 free-response questions. For the multiple-choice questions, respond with the letter of the correct choice. Explanations are not required. For short-response questions, just give a final answer, as asked for. Again, no explanation is required. For the free-response questions, type or write up your full solutions. Submit your response via the provided form or to

iPhOOContest@gmail.com

with your full name, member code, age and grade. If you are not in high school, state your educational status. Note that this will not affect your score. For each multiple-choice question, a correct answer will earn you 3 points, an incorrect answer will be penalized by 1 point, and leaving it unanswered will give you 0 points. For each short-answer question, you get 6 points for a correct answer and 0 points otherwise. For each free-response question, depending on your steps and correctness of your work, you will get a score between 0 and 10 points, inclusive. Thus, the maximum possible score is 125. The top 5 will receive prizes.

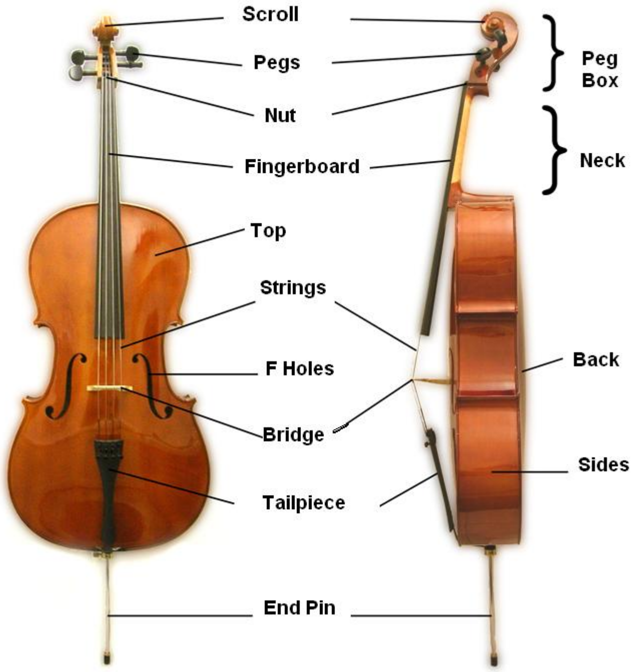
Although you have two weeks for this, you should be able to complete this entire set within 2 hours.

Multiple-Choice Questions.

1. If v is the speed of sound, ϵ_0 is the permittivity of free space, and μ_0 is the permeability of free space, what are the SI units of $\epsilon_0 \mu_0 v^2$?

   (A) $\frac{\text{m}}{\text{s}}$ **(B)** $\frac{\text{s}}{\text{m}}$ **(C)** $\frac{\text{m}^2}{\text{s}^2}$ **(D)** $\frac{\text{s}^2}{\text{m}^2}$ **(E)** None (unitless)

2. Shown above is a diagram with labels of the parts of a cello[1], a classical music instrument. Which of the following parts is closest to a cello's center of mass, assuming the end pin is extended?



   (A) Pegs
   (B) Fingerboard
   (C) Top
   (D) Bridge
   (E) Tailpiece

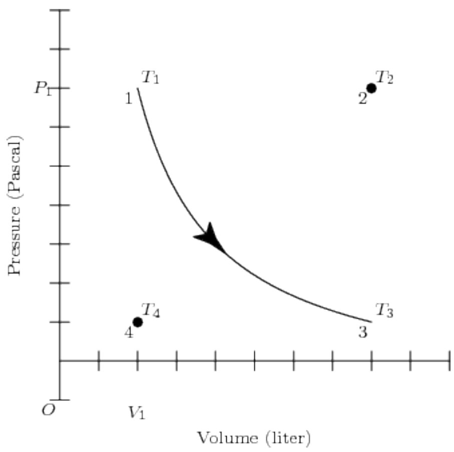
[1]Credits to Wikipedia

3. Josh and Jayne are on a spinning turntable that is rotating at a constant frequency. Jayne is sitting at the edge of the turntable. Josh is sitting between Jayne and the center of rotation. What of the following quantities is greater for Jayne than it is for Josh?

   I. Linear velocity
   II. Tangential acceleration
   III. Linear acceleration

   (A) I only
   (B) II and III
   (C) I and III
   (D) I, II, and III
   (E) none

4. An ideal gas is initially in a state that corresponds to point 1 on the graph below, where it has pressure P_1, volume V_1, and temperature T_1. The gas undergoes an isothermal process represented curve shown, which takes it to a final state 3 at temperature T_3. If T_2 and T_4 are the temperatures the gas would have at the states corresponding to points 2 and 4, and $T_1 = T_3 = T$, which of the following statements is true?



   (A) $T < T_4 < T_2$
   (B) $T_4 < T < T_2$
   (C) $T_4 < T_2 < T$
   (D) $T_2 < T < T_4$
   (E) none of the above

5. Assume a right-handed coordinate system. A negative charge moving in the $+\hat{\mathbf{x}}$ direction is deflected towards the $+\hat{\mathbf{y}}$ direction. What is the direction of the induced magnetic field?

(A) $+\hat{\mathbf{z}}$
(B) $-\hat{\mathbf{z}}$
(C) In the xy-plane
(D) In the yz-plane
(E) In the xz-plane

Short-answer.

1. A man of mass m is standing in an elevator. The elevator is accelerating downward at $\frac{g}{k}$, where $k > 1$ and g is the gravitational acceleration. If the force on the man by the elevator's floor is jmg, where j is a constant and $k = \frac{a}{b+cj}$, where $\gcd(|a|, |b|, |c|) = 1$, what is $a + b + c$? If k cannot be expressed in this form, say so.

2. Two charged spheres are kept at a great distance from each other. Sphere A of radius r has charge $+20q$ and sphere B of radius $2r$ has charge $-14q$. A neutral conducting sphere, sphere X of radius $3r$, is then touched upon the sphere A, then sphere B, and then sphere A again. If q_A, q_B, and q_X are the charges on spheres A, B, and X, respectively, after these contacts, what is the ordered triple $\left(\frac{q_A}{q}, \frac{q_B}{q}, \frac{q_X}{q}\right)$? Do not ignore signs!

3. In a binary star system in which two stars orbit each other about their center of mass, the mass of one star is $20m$ while the mass of the other star is m. In another binary star system, the masses of the stars are $14m$ and m. The fixed distance between the stars in the both cases are the same. Let v_1 be the velocity of the $20m$ mass. Let v_2 be the velocity of the $14m$ mass. Find the exact value of $\frac{v_1}{v_2}$.

4. Bradan drops a ball from a height of 1 meter above the ground. Jeff starts a camera which takes a picture of the ball every 250 ms, with the first click occurring at the moment Bradan lets go of the ball. Let the ball's height about the ground at the nth click, in meters, be h_n. For example, $h_1 = 1$. Compute the value of $\frac{h_3+h_4+h_5}{h_2}$.

   Assumptions

   - If the ball makes a collision with the ground, it will be perfectly elastic.

5. Josh has an infinite supply of solid cylinders all of different dimensions but of the same uniform volume density ρ. He arranges them in a row on his infinite table in descending order of heights and notices that the radius of the first cylinder is r, the radius of the second cylinder is $\frac{r}{2}$, etc., with the radius halving every time. Simultaneously, he realizes that the height of the first cylinder is h, the height of the second cylinder is $\frac{h}{2}$, etc., with the height halving every time. For example, the radius and height of the third cylinder are $\frac{r}{4}$ and $\frac{h}{4}$, respectively.

   Josh then stacks the cylinders on top of each other so that the axes form one line. In other words, the figure is completely symmetric across all axes of the cylinder. He stacks them so that every cylinder has a smaller radius and height than the cylinder below it. Assume he completes the infinite stack on his table, how high above the table is the center of mass of the system?

6. Consider the same set of cylinders as in the previous question. Now Josh puts the cylinders up against a vertical wall and decks up the cylinders such that the lateral surfaces of all the cylinders touch the wall. What is the distance between

the center of mass and the point at which the bottommost cylinder touches the ground and the wall.

7. A very long rod of length L has uniform density per unit length. The rod is now left to swing as a pendulum with oscillations of small angle displacement with the pivot at some point on the rod. What is the shortest possible temporal period of the pendulum's oscillations, in terms of L and g?

8. Josh has an infinite supply of capacitors of capacitance $1\,\mu$F each. He arranges them in a circuit in the following manner. First, he makes an infinite number sub-circuits such that the kth sub-circuit contains k parallel sets of k capacitors in series. For example, for $k = 1$, it is just a single capacitor:

$$-||-$$

He does this for all integers $1 \le k \le n$ and puts the n sub-circuits in series. What is the smallest value of n such that the equivalent capacitance of the entire circuit is less than $1\,$nF?

9. What is the magnitude of the electric field at the tip of a hollow cone of height h, base radius r, and uniform surface charge density σ?

10. Suppose that a gene, *genX*, is its own activator; i.e. the protein product, X, of the *genX* promotes the transcription of the gene's messenger RNA. Moreover, the protein product is only active as a transcription factor when it is bound to itself in dimer form. The dissociation constant of the dimer is K_2, and the binding constant of the dimer to the mRNA's promoter region is K_{act}. The maximum transcription rate of the mRNA is $\epsilon_{\text{txn}} + \alpha_{\text{txn}}$, where ϵ_{txn} is the leakage transcription rate, i.e. some basal mRNA production that takes place even in the complete absence of the dimer. Moreover, each mRNA is translated into protein at some rate α_{tln}, the mRNA decays at some rate β_{deg}, and the protein is diluted by cell growth at the rate λ.

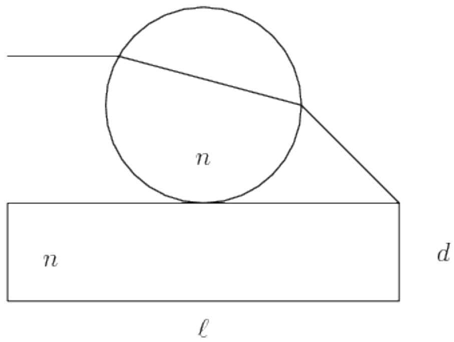
    At fast growth rates, λ, there is only one admissible level for the concentration of X. However, as growth slows, the system undergoes a bifurcation and two distinct stable concentrations are possible. Find the critical value of the growth rate, λ_c^+, at which this phase transition occurs (coming from the fast growth regime). Use $\epsilon_{\text{txn}} = 0.03$ mM/hr, $K_{\text{act}} = 0.1$ mM, $K_2 = 2000$ mM, $\alpha_{\text{txn}} = 15$ mM/hr, $\beta_{\text{deg}} = 10$ hr^{-1}, and $\alpha_{\text{tln}} = 10$ mM/hr. You should submit a numerical answer.

    Assumptions

    - mRNA dynamics are much faster than protein dynamics ($\lambda \ll \beta_{\text{deg}}$)

Free-response.

Problem 1 (Left-right-down-uptics). In the following problem, a solid sphere of radius r made of a material with index of refraction n sits at rest on a solid table made of the same material. The table is of length ℓ and of height d. It is simply a prism and does not have legs. The diagram shows a side view in the specific case of part **(b)**. A light beam is pointed horizontally at the sphere from the left at a height $h > r$; it penetrates the sphere, and exits, as shown. Answer the questions below and show all your work. Express all answers in terms of n, r, and/or d, as necessary.

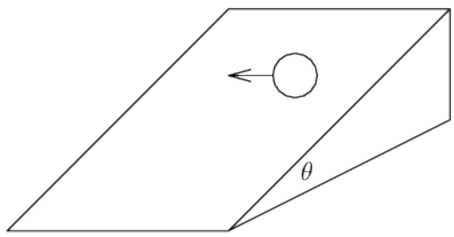


(a) Find the height h, such that the exiting ray is parallel to the table. If this is not possible, prove it.

(b) Assume the sphere is "in the middle of the table" $-$ that is, the point of contact is at the midpoint of the segment that connects the two ends of the table. Find the height h such that the light beam meets the table at the rightmost end, as shown in the diagram. If this is not possible, prove it.

(c) Continuing with the same scenario, find the height h such that the ray meets the ground at the bottom-right-hand corner of the table. If this is not possible, prove it.

Problem 2 (Rolling to the side). An inclined plane is set at an angle θ above the ground. There is a ball that touches the ramp. At time $t = 0$, the ball is at a distance L from the bottom of the incline and it is at rest. At that moment, it is given an initial velocity to the side (parallel to the bottom edge of the wedge) of v_0 and the ball rolls without slipping.



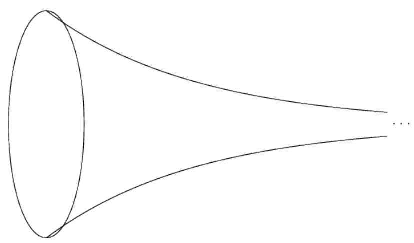
Let P be the point at which the ball hits the ground at the bottom of the wedge. Let Q be the point at which the ball would have hit the ground if it were just let to roll down the incline. Find, with proof, the distance PQ. Express your final answer in terms of v_0, L, θ, and/or any physical constants, as necessary. Assume that the wedge is infinitely wide, so the ball does not fall off the ramp.

Problem 3 (Resistor Lab). Josh is making resistors out of metal wire. Assume the metal he uses has resistivity ρ.

(a) Josh is still a student so he simply makes a cone of wire with radius b. Prove that his resistor has infinite resistance.

(b) Josh now chops a smaller cone off the top of his current cone to make a truncated cone of height L and radii a and b, with $a < b$. Find, with proof, the resistance of his frustum-cone resistor.

(c) Josh is starting to get the hang of this. He wants to be creative now, so he makes an infinitely long resistor of wire in the shape of a curved cone! The base is still a circle with radius b. However, now, instead of using a linear surface, the lateral sections of his cone are exponential curves, as shown in the figure.



The exponential curve is such that at a distance of ℓ from the base, the cross-sectional radius is
$$r(\ell) = b \cdot \exp(-\ell).$$
Does this new resistor have an infinite resistance? If not, find, with proof, the magnitude of its resistance.

Problem 4 (Gravitational Lensing). At one point, the earth, the sun, and a star are collinear, and are contained in the xy-plane. The earth is at the point $(-r, 0)$, the sun is at $(0, 0)$, and the star is at $(R, 0)$. Due to a certain type of gravitational lensing, the light from the star is scattered to the line $x = 0$ in straight lines that make an angle of θ with respect to a horizontal axis and is bent to the earth with straight lines that make an angle of α with respect to a horizontal axis. This gravitational lensing occurs in a manner such that $\theta + \alpha = k$, a constant. Consider the image of the star, which is a ring.

(a) Is the image real or imaginary? Explain.

(b) Find, with proof, the radius of the ring.

(c) Find, with proof the coordinates of the center of the ring.

Problem 5 (Electron in a pyramid). An electron is inside in a square pyramid each of whose edges has length a. Estimate the minimum pressure on one face of the pyramid. Note that there is no one correct answer to this question. However, your estimate, which should be in terms of m, the mass of the electron, a, and/or other constants, as necessary, should be reasonable. Again, show all your work so we know your estimate is acceptable.