IPhOO: Contest 1

Thanks to the following individuals:

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Problem 1 (Ahaan Rungta)**.** A construction rope is tied to two trees. It is straight and taut. It is then vibrated at a constant velocity v_1. The tension in the rope is then halved. Again, the rope is vibrated at a constant velocity v_2. The tension in the rope is then halved again. And, for the third time, the rope is vibrated at a constant velocity, this time v_3. The value of $\frac{v_1}{v_3} + \frac{v_3}{v_1}$ can be expressed as a positive number $\frac{m\sqrt{r}}{n}$, where m and n are relatively prime, and r is not divisible by the square of any prime. Find $m + n + r$. If the number is rational, let $r = 1$.

Problem 2 (B. Dejean)**.** One hundred billion light years from Earth is planet Glorp. The inhabitants of Glorp are intelligent, uniform, amorphous beings with constant density which can modify their shape in any way, and reproduce by splitting. Suppose a Glorpian has somehow formed itself into a spinning cylinder in a frictionless environment. It then splits itself into two Glorpians of equal mass, which proceed to mold themselves into cylinders of the same height, but not the same radius, as the original Glorpian. If the new Glorpians' angular velocities after this are equal and the angular velocity of the original Glorpian is ω, let the angular velocity of the each of the new Glorpians be ω'. Then, find $\left(\frac{\omega'}{\omega}\right)^{10}$.

Problem 3 (B. Dejean and Ahaan Rungta)**.** A rigid (solid) cylinder is put at the top of a frictionless 25°-to-the-horizontal incline that is $3.0\,\mathrm{m}$ high. It is then released so that it rolls down the incline. If v is the speed at the bottom of the incline, what is v^2, in $\mathrm{m}^2/\mathrm{s}^2$?

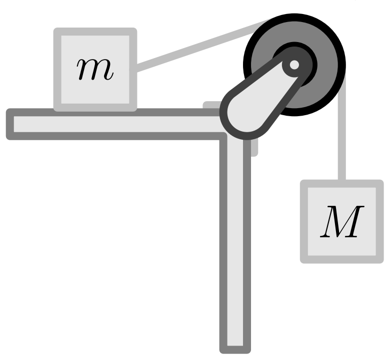

Figure 1: Problem 4

Problem 4 (Ahaan Rungta)**.** A pulley system of two blocks, shown above, is released from rest. The block on the table, which has mass $m = 1.0\,\mathrm{kg}$ slides after the time of release and hits the pulley to come to a dead stop. There was originally a distance of $1.0\,\mathrm{m}$ between the block and the pulley, which the block fully covers during the slide. From the time of release to the time of hitting the pulley, the angle that the rope above the table makes with the horizontal axis is a, nearly constant, $10.0°$. The hanging block has mass $M = 2.0\,\text{kg}$. The table has a coefficient of friction of 0.50 with the block that sits on it. The pulley is frictionless. Also, assume that, during the entire slide, the block never leaves the ground. Let t be the number of seconds in takes for the 1.0-m slide. Find $100t$, rounded to two significant figures.

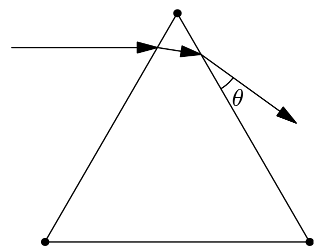

Figure 2: Problem 5

Problem 5 (Ahaan Rungta)**.** The cross-section of a prism with index of refraction 1.5 is an equilateral triangle, as shown above. A ray of light comes in horizontally from air into the prism, and has the opportunity to leave the prism, at an angle θ with respect to the surface of the triangle. Find θ in degrees and round to the nearest whole number.

Problem 6 (B. Dejean)**.** A particle with charge $8.0\,\mu\text{C}$ and mass $17\,\text{g}$ enters a magnetic field of magnitude 7.8 mT perpendicular to its non-zero velocity. After 30 seconds, let the absolute value of the angle between its initial velocity and its current velocity, in radians, be θ. Find 100θ.

Problem 7 (B. Dejean)**.** Ancient astronaut theorist Nutter B. Butter claims that the Caloprians from planet Calop, 30 light years away and at rest with respect to the Earth, wiped out the dinosaurs. The iridium layer in the crust, he claims, indicates spaceships with the fuel necessary to travel at 30% of the speed of light here and back, and that their engines allowed them to instantaneously hop to this speed. He also says that Caloprians can only reproduce on their home planet. Call the minimum life span, in years, of a Caloprian, assuming some had to reach Earth to wipe out the dinosaurs, T. Assume that, once a Caloprian reaches Earth, they instantaneously wipe out the dinosaurs. Then, T can be expressed in the form $m\sqrt{n}$, where n is not divisible by the square of a prime. Find $m + n$.

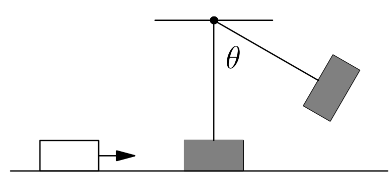

Figure 3: Problem 8

Problem 8 (Ahaan Rungta)**.** A block of mass $m = 4.2$ kg slides through a frictionless table with speed v and collides with a block of identical mass m, initially at rest, that hangs on a pendulum as shown above. The collision is perfectly elastic and the pendulum block swings up to an angle $\theta = 12°$, as labeled in the diagram. It takes a time $t = 1.0$ s for the block to swing up to this peak. Find $10v$, in m/s and round to the nearest integer. Do not approximate $\theta \approx 0$; however, assume θ is small enough as to use the small-angle approximation for the period of the pendulum.

Problem 9 (B. Dejean)**.** Bob, a spherical person, is floating around peacefully when Dave the giant orange fish launches him straight up 23 m/s with his tail. If Bob has density 100 kg/m^3, let $f(r)$ denote how far underwater his centre of mass plunges underwater once he lands, assuming his centre of mass was at water level when he's launched up, where r is the radius of the sphere. Find $\lim_{r\to 0}(f(r))$. Express your answer is meters and round to the nearest integer. Assume the density of water is 1000 $\mathrm{kg/m^3}$.

Note: Calculus is not required for this problem. The limit just stands as an infinitesimal approximation.

Problem 10 (Trung Phan)**.** Two masses are connected with spring constant k. The masses have magnitudes m and M. The center-of-mass of the system is fixed. If $k = 100$ N/m and $m = \frac{1}{2}M = 1$ kg, let the ground state energy of the system be E. If E can be expressed in the form $a \times 10^p$ eV (electron-volts), find the ordered pair (a, p), where $0 < a < 10$, and it is rounded to the nearest positive integer and p is an integer. For example, 4.2×10^7 should be expressed as $(4, 7)$.